

# Determining Coefficient of Discharge to Compare Coefficients of Resistance for Different Coarse Aggregate Beds

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## ABSTRACT

The paper is an investigation to determine the coefficient of discharge for different notches and weirs with coarse aggregate beds and then comparing Chezy's, Darcy Weisbach's and the Manning's coefficients. Prior to that, the notches had to be designed and fabricated, then experiments were conducted. Six methods were used for this experiment. The open channel on the hydraulic bench is intended to depict an actual open channel flow scenario, but on a small scale, steady flow was used for these experiments. From this investigation, it was confirmed that discharge coefficient for all notches and weirs ranges between 0.57-0.9. The bed of a channel, in this case, coarse aggregates was found to have an effect on discharge coefficient, though it is said to be insignificant. Manning's  $n$  and Darcy-Weisbach's  $f$  were found to decrease with increase in discharge and velocity.

**Keywords:** Open Channel Flow, Flow Resistance, Discharge Coefficient

## I. INTRODUCTION

The purpose of this experimental research is to determine discharge coefficients for different types of weirs and notches and then comparing coefficients of resistance for different aggregate size beds. These coefficients of resistance are experienced when a liquid is flowing through a channel. Liquids flow from one point to another, either naturally using the force of gravity or by the use of constructed conveyance structures. In general, the coefficient of discharge is ratio of mass flow rate at the discharge end to that at a particular point along the channel or at the end of the channel. They are basically 2 types of liquid flow which are closed conduits whereby the top part of the channel where the flow is taking place is closed, for example pipes and open channel flow referred as the flow of liquid exposed to the atmosphere like rivers, canals and tunnels.

Open channel flow includes a very wide range of flows, from flows occurring in natural channels like rivers to gutters along residential streets. When a liquid flows in an open channel, the free space is in actual fact an interface between two fluids of different density. In the case of the atmosphere, the density of the air is less than

that of a liquid like water for example and the pressure is constant. On the other hand, when a fluid is flowing, more often than not, the motion is usually caused by gravitational effects and the pressure distribution within the fluid is generally hydrostatic. Open channel flow is almost always turbulent and is not affected by surface tension, however in many cases of practical importance, flows of this type are density-stratified. For this paper, steady flow was used, though very rare in natural streams, it is the condition that is frequently assumed in open channel flow conditions.

These channels play an integral part in the lives of humans, thus the need to fully understand them so that the natural ones can be fully manipulated, at the same time creating man made designs that can change human lives for the better.

## II. METHODS AND MATERIAL

### A. Literature Review

Coefficient of discharge is defined as the ratio of the mass flow rate at the discharge end of the channel to the theoretical discharge rate. There are numerous ways to determine the discharge coefficient in a hydraulics

laboratory. The common ones being the use of weirs, orifice, flumes, sluice gates and notches. In practise, the structures used for flow measurements should be accurate precise, economical and also easy to use (installation, operation and maintenance). Weirs allow flow water to be diverted to a structure that is calibrated, thus allowing flow rates to be measured as a function of depth of flow through the structure. In application, among other different types of weirs, sharp-crested weirs have been the most commonly used type of weir in open channel flow, this includes notches. Triangular, rectangular and trapezoidal seem to be the more frequently used than the other type of weirs. For discharge measurements, a triangular notch with a small angle is found to be more accurate. However it can only measure low discharges [20]. This is because when a notch is used in open channel flow, the flow starts at a point and both discharge and flow increase as a function of depth. This will in turn spread out the low discharge end of the depth discharge curve, thereby increasing the accuracy. When high discharges are to be measured by single sharp crested weirs, backwater effects may end up affecting the structures located upstream of the weir, thereby reducing the accuracy.

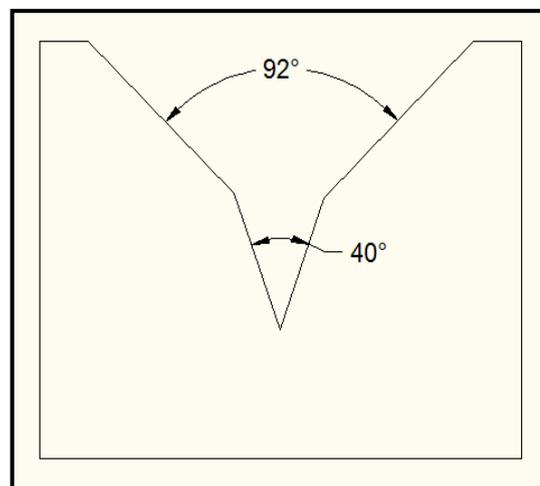
When discharge measurements are to be undertaken with a reasonable sensitivity over a wide range of flows, the use of compound sharp crested weirs could be the appropriate solution [20].

To improve accuracy of flow measurements, modifications to the existing apparatus have taken place after some extensive research by the investigators. Consequently new types compound notches have been developed along with new formulas. Some of the modified notches are a combination of two separate notches or a combination of two shapes, like the one shown in the figure below. Figure 1 below shows a typical example of a compound notch combining a rectangular notch and a triangular notch. (USA Forest Services 1999)



**Figure 1:** Compound notch, suppressed rectangular with  $90^\circ$  triangular notch

However the compound weir shown in the image above has a major disadvantage when discharge begins to exceed the capacity of the triangular notch. When this happens, thin sheets of water will begin to spill over to the wide horizontal crests thereby causing discontinuity in the discharge curve [8]. In irrigation canals, the most commonly used compound weirs is a combination of a rectangular notch and a triangular notch with a small angle. Previous research suggests that the accuracy of flow measurements of this type is average when measurements are to be undertaken in the transition region between the two parts, ie; between the two shapes. Therefore to overcome this problem and also to measure discharge more accurately, research on using a combination of two triangular notches with different angles has been ongoing [20]. Example of such is shown in Figure 2 below.



**Figure 2:** Typical Cross section of compound weirs [19].

According to some researchers, the head over the weir can be adjusted to eliminate the effects of lateral and vertical contractions, though the applicability of this technique is limited [9], [17], [23]. As research progressed, a correction factor  $K$  was introduced to the head over the weir after the consideration of surface tension and viscous effects [3]. The value of the discharge coefficient is highly dependent on the angle of the notch for fully developed flow while for partially developed flow it is dependent on many other various factors. British Standard Institution recommends that the head over the weir has to be adjusted for partially and fully contracted types of flow. Most sources show similar curves for discharge coefficient and  $K$  without providing equations for the two. [1] used a curve fitting programme to develop the

formulas below, both formulas are related to the angle of the notch.

$$C_d = 0.6072 - 0.000874 \theta + 6.1 \times 10^{-6} \theta^2 \quad (1)$$

$$K = 4.42 - 0.1035 \theta + 1.005 \times 10^{-3} \theta^2 - 3.24 \times 10^{-6} \theta^3 \quad (2)$$

The tables below show typical theoretical values of discharge coefficients for different types of weirs [27].

**Table 1:** Typical values of Discharge coefficients

Type of weir/flume	Max $C_d$	Min $C_d$
Broad crested weir	0.6	0.9
Sharp crested weir	0.57	0.9
Crump weir	0.57	0.9
Overshot weir	0.6	0.9
Venturi flume	0.6	0.9

**Table 2:** Triangular weirs coefficient of discharge

Head (cm)	Weir Angle (degrees)				
	22.5	30	45	60	90
15.24	0.611	0.605	0.596	0.590	0.584
30.48	0.593	0.590	0.583	0.580	0.576
45.72	0.586	0.583	0.578	0.575	0.572
60.96	0.583	0.580	0.576	0.573	0.571
76.2	0.580	0.578	0.574	0.572	0.570

The Manning's formula, developed in 1889 by Robert Manning, an Irish engineer, the Manning's formula has recently been modified to its present well known form:

$$V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \quad (3)$$

This formula developed from different formulas based on Bazin's experimental data then it was verified by 170 observations. Because of its simplicity and clarity, the Manning's constant is the preferred constant of friction in practical applications. Consequently it has become the most widely used of all uniform flow formulas for open channel flow computations.

When applying Manning's formula, complications may arise in determining the coefficient of roughness  $n$ , this is because there is no exact method for selecting the  $n$  value. Currently, with present knowledge, the process of selecting  $n$  actually means to estimate the resistance of flow in a given channel which will not be accurate, though experienced engineers can use their experience to obtain a more accurate value of  $n$ , unlike novice engineers. It would be naive for engineers to assume that the value of  $n$  is the same along the whole channel as this is highly unlikely. In actual fact, the value of  $n$  is extremely variable and is dependent upon a range of design factors.

According to [24], these are the factors that determine Manning's coefficient. Please note that these factors are dependent on each other and the effect of a certain factor may also be mentioned be in another factor: surface roughness, vegetation, channel irregularity, channel alignment, silting, scouring, obstruction, stage, discharge and seasonal change.

The Chezy's formula, regarded as the first uniform flow formula was developed by Antoine Chezy in 1769 is given below.

$$V = C\sqrt{RS} \quad (4)$$

Mathematically the Chezy's formula can be derived from 2 assumptions, one made by Chezy and the other made by Brahms in 1754. The one made by Chezy states that the force resisting the flow per unit area of the stream bed is proportional to the square of the velocity; that is the force is equal to  $KV^2$  where  $K$  is a constant of proportionality.

A simple derivation developed by [11] is given below. The surface of contact of flow with the stream bed is equal to the product of the wetted perimeter and the length of the channel reach. This implies that:

$$\text{Force resisting the flow} = KV^2PL \quad (5)$$

Second assumption is a basic principle in uniform flow which is believed to have been developed by Brahms. This principle states that in uniform flow the effective component of gravity-force causing the flow must be equal to total force of resistance. The effective gravity-force component is parallel to the channel bottom and equal to:

$$W A L \sin \theta = W A L S \quad (6)$$

Hence:

$$w A L S = K V^2 P L \quad (7)$$

Let  $A/P = R$  and let  $\sqrt{w}/K$  be replaced by a factor  $C$ , then the previous equation can be replaced by the Chezy's formula:

$$V = C\sqrt{RS} \quad (8)$$

Darcy-Weisbach's frictional factor, though commonly used in pipes, this formula is also used in open channel flow computations to calculate the frictional factor of the wetted perimeter in the channel. Developed by Weisbach in 1845 for pipes, it was later refined by Darcy. In actual fact, the American Society of Civil Engineers Task Force on frictional factors supported the use of this formula in open channel flow [23]. Consequently the formula has been applied to open channels in its general form given below:

$$C = \sqrt{\frac{8g}{f}} \quad (9)$$

[11] classified elements of flow resistance into four components:

- i) Surface or skin friction.
- ii) Form resistance or drag.
- iii) Wave resistance from free surface distortion.
- iv) Resistance associated with local acceleration or flow unsteadiness.

After some intensive research he came up with the formula below which incorporates all the elements using the Darcy-Weisbach's formula.

$$f = F(R, K, \Omega, N, F, U) \quad (10)$$

Where:

$f$  = Darcy-Weisbach's frictional factor

$R$  = Reynolds number

$F$  = Froude number

$F$  = Function

$K$  = Relative roughness expressed as  $k_s/R$  where  $k_s$  is the equivalent of wall surface roughness and  $R$  is the hydraulic mean radius of the channel

$\Omega$  = Cross sectional geometrical shape

$N$  = Non-uniformity of channel in both profile and plan

$U$  = Degree of flow unsteadiness

Developed in 1883, Reynolds number is defined as a dimensionless parameter that gives a measure of the ratio of inertial forces to viscous forces. The number is then used to quantify the flow conditions. Froude number is the ratio of characteristic velocity to gravitational wave velocity. It is also a dimensionless parameter.

The most common source for  $f$  is the Moody diagram.

## B. Previous Work

Research by previous authors clearly shows that Manning's roughness coefficient  $n$  is the commonly used coefficient in the design of open channels. A formula to show the relationship between the three coefficients is given below.

$$\sqrt{\frac{f}{8}} = \frac{n}{R^{\frac{1}{6}}} \frac{\sqrt{g}}{1} = \frac{\sqrt{g}}{c} = \frac{\sqrt{gRS}}{V} \quad (11)$$

From equation (11) it can be noted that from knowing the value of one of the coefficients, the corresponding coefficients can be determined using the formula. This equation is derived from the coefficient formulas and it shows that the three are in similar form and are interchangeable (James et al., 2001). Generally, different equations are used for computations in different scenarios and the required coefficients are estimated in different ways. Equation (11) can also be applied to momentum or energy resistance coefficients for a point along the channel and also a cross section of the channel. When used in the form above, the equation clearly shows that there is no clear theoretical advantage of one coefficient over the others therefore a comparison of the three coefficients and formulas from a practical aspect may be useful since it is practically impossible for there to be no difference among the three [6].

All the equations that have been stated in this paper are only applicable if the flow is steady. Steady uniform flow is a flow in an open channel, is flow in an open channel where the depth of flow does not change, or the flow is assumed to be constant during the time interval under consideration. [14]

Traditionally the Darcy-Weisbach's  $f$  has the advantage of being directly related to fluid mechanics by scientists and engineers worldwide to the point whereby

sometimes it may be misquoted as a theoretical coefficient. The Chezy formula is the most simple to use and was the first to be developed. On the other hand Manning's  $n$  is assumed to be nearly constant and is almost independent of flow depth and when the Reynold's number  $R$ , which determines whether the flow is laminar or turbulent is highly turbulent over a rigid rough surface. The most reliable source to find  $f$  is the Moody diagram, developed by Moody using  $f$ ,  $R$  and roughness relative roughness. [23] developed tables for Manning's  $n$ , this the most common and reliable source to estimate the values of  $n$ . For Chezy's  $C$  there has not been any recognised table to date.

In the field of fluid mechanics,  $f$  is sometimes associated with the concept of shear-momentum. It is rare for other engineers in various engineering disciplines other than hydraulic engineering to consider  $f$  as an energy loss coefficient. Traditionally  $f$  is regarded as a point value that is related to the distribution of velocity and discharge of channel, though at times some hydraulic engineers extend it to cross section or reach values and they may also consider it as energy loss coefficient [6].

The determination of resistance coefficient values by Manning used data obtained from the field followed the head loss energy concept that is applied to channel reaches. (Manning's 1891 article]. Channel wall resistance was also mentioned. In hydraulic engineering  $n$  has been used to calculate values for channel reach energy loss coefficient values. Thus, it seems as if it is more appropriate to use Darcy- Weisbach's frictional factor for point resistance while Manning's  $n$  can be used for cross sectional and reach resistance coefficients. Field experience from professional engineers suggests that  $n$  is a simpler coefficient that can accommodate the effects of other parameters.

Another form of Darcy-Weisbach formula given in equation (10) can also be applied to the Manning's resistance coefficient in the form of:

$$\frac{n}{k_s^{1/6}} \quad (12)$$

When used this way, equation (12) will replace the  $K$  in equation (10) which is the relative roughness. [12] was the one who went on to develop what is now known as the Moody diagram. The Moody diagram is mainly used in the design of pipes and is applicable for steady uniform flow in straight, constant diameter rigid pipes. From equation (10) only two of the six parameters given

in the equation are taken into consideration that is the Reynolds number  $R$  and the relative roughness  $k_s/R$  for Nikuradse type dense random surface. In practise, the six parameters in equation (10) and the four elements that resist flow in channels interact in a nonlinear manner such that any linear relationship is merely based on assumption. For this reason, in open channel flow computations, equations (3), (8) and (9) are commonly used.

When the Moody diagram was being developed, equivalent roughness  $k_s$  was used. Values of  $k_s$  were obtained using the relationship shown in equation (10).

### C. Research Methodology

For this investigation, a hydraulic bench, open channel, three weirs and three notches were used along with different sizes of coarse aggregates. The table shows the specifications of the hydraulic bench and open channel.

**Table 3:** Channel Specifications of Hydraulic Bench and Open channel

	Open Channel	Hydraulic bench
Length	2.5 meters, testing section 1 meter	0.9 meters, full bench used to test
Width	0.079 meters	0.25 meters

Once the discharge coefficients and the resistance factors were obtained an analysis was then done. Values obtained on the open channel were also then compared to those obtained on the hydraulic bench. The data obtained from these experiments may only be applicable to small open channels like small rivers, canals and streams. Steady flow will be used in this research and the slope will be kept at a constant value of one. Most of the previous works relevant to this research paper have mainly focussed on Manning's and Chezy's coefficient as they are the main coefficients used in open channel hydraulics, Darcy-Weisbach is more common for pipes. However this text will further document the relationship between the three coefficients that is Chezy's, Manning's and Darcy-Weisbach's.

According to [19] notches tend to be more accurate than weirs. This is because when water flows over a notch, it starts at a point where both the discharge and width of flow gradually increase as a function of the depth. This will then result in dispersing the low discharge end of the depth discharge curve, thereby allowing a more accurate value of discharge to be obtained. In these experiments steady flow was used.

### III. RESULTS AND DISCUSSION

The area of discharge coefficients has been extensively investigated. At present moment, a few tables of expected discharge coefficients have been published, though they might be small deviations, they are usually within the same range. A comparison of the results obtained from this investigation and those published by (Water Resources Engineering, M Kent Loftin,1999) is given in the table below.

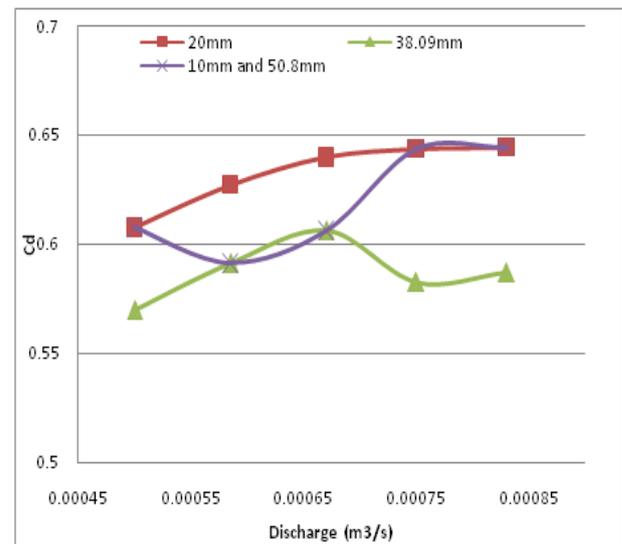
**Table 4:** Expected and Observed  $C_d$  values

	Min $C_d$	Max $C_d$	Observed $C_d$
Crump	0.57	0.9	0.817
Sharp crested	0.57	0.9	0.633
Overshot	0.6	0.9	0.513
Rectangular	0.57	0.9	0.582
$60^\circ$	0.57	0.9	0.585
$90^\circ$	0.57	0.9	0.638

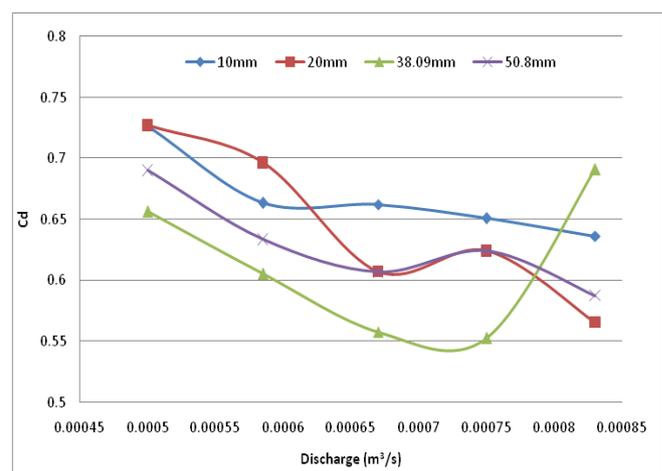
Factors that affect the discharge coefficients are given below:

- Whether the tests were conducted in the laboratory or in a natural set up.
- Size of the apparatus used.
- Type of flow used
- Slope during the experiments.
- Range of flow used for the testing.
- The terrain of the channel.

Literature relating discharge coefficient to channel bed could not be obtained. This is because of the complex nature on the subject. When aggregates are on the bed of a channel, a lot of factors need to be considered, size, spacing, arrangement, and friction to name a few. In the analysis of aggregate size on coefficient of discharge, it will be difficult, if not impossible to accurately determine the effect of size only on the discharge coefficients, considering the fact that it is not only size that has an effect. Below is a graph showing the variation of discharge coefficient and aggregate size.



**Figure 2:** Graph showing discharge coefficient values for different aggregate sizes

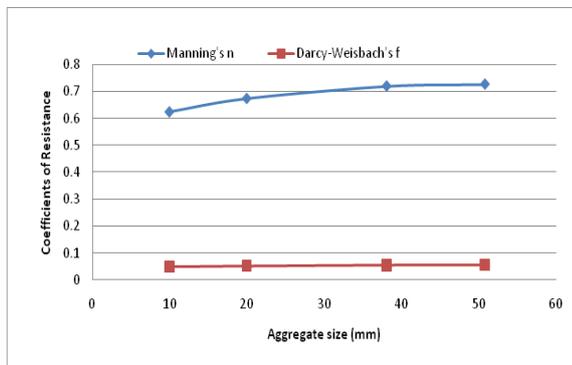


**Figure 3:** Graph showing discharge coefficient values for different aggregate sizes

Figure 2 shows results obtained using a sharp crested weir while in Figure 3 a  $60^\circ$  notch was used. From these two graphs it difficult to conclude the effect of aggregate

size on the discharge coefficient. However on a closer look at the trend it would seem as if the graph would be that of simple harmonic motion and its decreasing. The current formula to calculate discharge coefficient does not accommodate for aggregate size as it is said to be negligible thus the lack of need to consider aggregate size in practice. In designs of high accuracy, it might be necessary to determine the actual effect of aggregate size. [16], concluded that a channel does not have an effect on coefficient of discharge, implying the bed of channel may not affect the discharge coefficient obtained for a particular weir on that particular bed. On the other hand, the aggregates are bound to have an effect on the flow of the water, Roberson and Wight (1973). The time taken by the water to flow along the channel will be increased because of the obstruction and backwater effects will inevitably be experienced due to the presence of the aggregates. For as long as there are no modifications to existing formulas to accommodate aggregate size, the real effect of the bed on discharge coefficient will remain elusive.

A typical graph of resistance coefficients against aggregate size shown below.

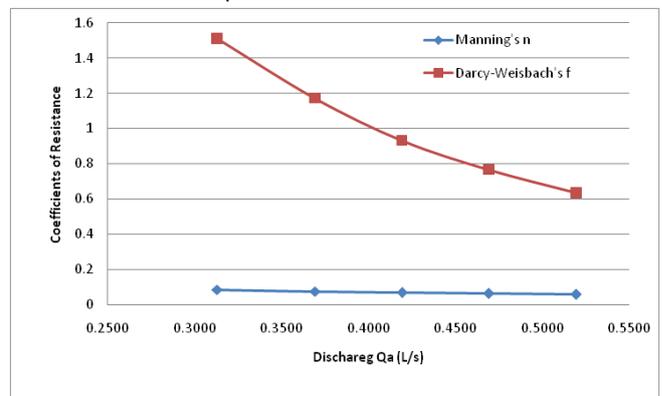


**Figure 4:** Graph of resistance coefficients against aggregate size.

From the graph above it is clearly evident that resistance in a channel increases with aggregate size. However discharge in a channel is directly proportional to velocity, consequently decreasing the resistance in an open channel. This is illustrated in figure 5. It is also worth mentioning the fact that when resistance is high and discharge is low, the difference between the Manning's  $n$  and Darcy-Weisbach's  $f$  is more than when resistance is low and discharge is high. This would mean that when there is high resistance, the difference between these two coefficients is substantial and when discharge

increases, the difference is reduced. Some authors use the formula below to determine Darcy-Weisbach's frictional factor.

$$C = \sqrt{\frac{2g}{f}} \quad (13)$$



**Figure 5:** Graph of resistance against discharge in an open channel

For this particular investigation, the difference between the two formulas is merely numerical since the major difference between the two formulas is based on gravity. This is because the setup of this experiment maintained a slope of one, implying that the likelihood of any gravitational influences was low.

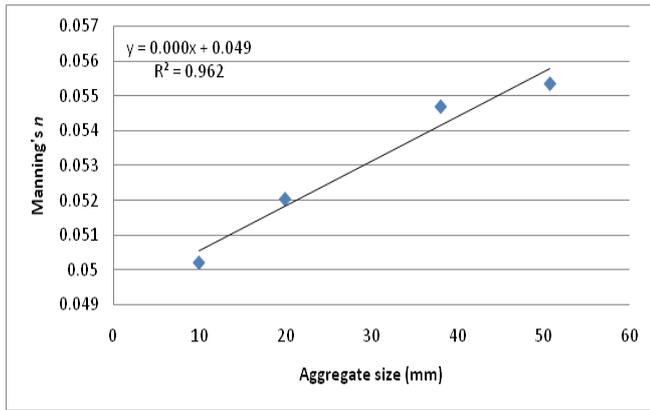
The density of aggregates affects the resistance in a channel. In this scenario, the aggregates on the hydraulic bench will definitely be higher than those in the open channel because of the difference in width. This basically means that the size of the channel will be a factor along with the density of the aggregates. In this case, the aggregates in the hydraulic bench had a higher density than those in the hydraulic bench.

A comparison of the average manning's  $n$  values of the four aggregate sizes was made with [22] table. The values obtained for this investigation were found to be in the same range, however Chow's table does not give Manning's values for aggregate sizes. Such a table is given below.

Table 5: Values of  $n$  for different beds

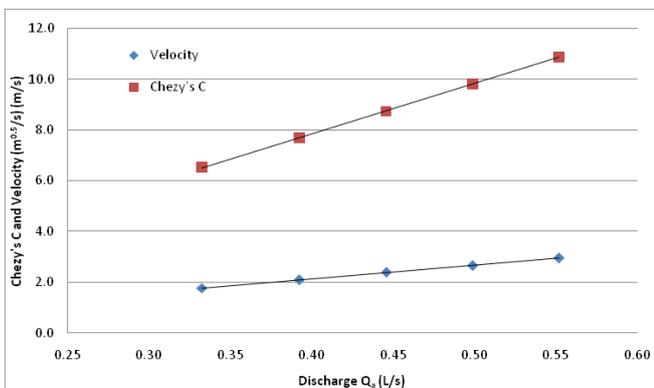
Aggregate size	Manning's n
10mm	0.050215
20mm	0.052047
38.09mm	0.054706
50.8mm	0.055366

The high dependence of Manning's  $n$  values on aggregate size is shown below by the high  $R^2$  value.

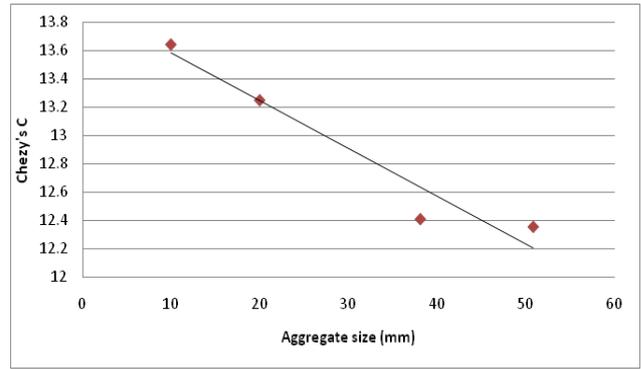


**Figure 6:** Graph of aggregate size against Manning's  $n$

As the size of aggregates increase, more friction and obstruction of the water in the channel is experienced. Though permeability of the bed increases due to the more and bigger gaps between the aggregates, resistance to flow will increase. The increase in back water effects will also be a major factor in increasing the resistance. On the other hand, the relationship between Chezy's  $C$  and velocity is directly a proportional one. This is illustrated Figure 7.



**Figure 7:** Variation of velocity components and discharge The velocity components are highly dependent on discharge and when discharge increases, velocity increases, thereby reducing the friction that is experienced in the channel. However as the aggregate sizes increases, Chezy's  $C$  decreases. This is evidenced in Figure 8.



**Figure 8:** Relationship of Chezy's  $C$  and aggregate size A comparison of figure 8 and figure 9 will show that Manning's  $n$  and Darcy Weisbach's  $f$  are inversely proportional to Chezy's  $C$ .

#### IV. CONCLUSION

Theoretically there does not seem to be any clear advantage of any formula over the others, therefore any comparison is made from a viewpoint [6]. The Chezy's formula is the most simple to use and also has the longest history. Darcy-Weisbach's  $f$ , though more popular for pipes can also be used in open channels and was recommended by United States Geological Survey. However Manning's  $n$  seems to be more popular. This could be because the reference tables are easily available and can be applied during the design of open channels. In 1967, Barnes came up with a picture book for Manning's  $n$  reference. The high repeatability of Manning's  $n$  values makes it more easier to use at the same time allowing more research to be done on it so that comparisons can be made. The Moody diagram presents a source for Darcy-Weisbach's  $f$ , but there is no recognised table or figure for Chezy's  $C$  values. This could be due to the low level of reproducibility of Chezy's  $C$  experiments.

The following conclusions were made from this investigation.

- i) Experiments to determine discharge coefficient were conducted on the hydraulic bench and open channel with different types of weirs and notches for different sizes of coarse aggregate beds, the following observations were found:
  - a) The head in an open channel increases with discharge.
  - b) Velocity increases with discharge.
  - c) Coefficient of discharge for all notches and weirs varies between 0.57-0.9.
- ii) The observations made with respect to resistance are given below:
  - a) Chezy's  $C$  is inversely proportional to Manning's  $n$  and Darcy-Weisbach's  $f$ .

- b) Manning's  $n$  and Darcy-Weisbach's  $f$  increase with increase in density and size of aggregates on the bed.
- c) Increase in velocity and discharge causes a reduction in resistance.

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