

# Some Generalised Characteristics of the Electro-dynamics of the Plasma Focus in Its Axial Phase: Illustrated by an Application to Independantly Determine the Drive Current Fraction and the Mass Swept-Up Fraction

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**Abstract** We describe the axial phase of the Mather plasma focus by two coupled equations of motion and circuit. We non-dimensionalised these equations resulting in two coupled equations which are characterised by only three scaling parameters  $\alpha$ ,  $\beta$  and  $\delta$  which are ratios of electrical to transit times, inductances and impedances respectively. The normalised current waveform, trajectory and speed profile are unique for each combination of  $\alpha$ ,  $\beta$ ,  $\delta$  which are the ratios of characteristic times (electrical discharge vs. axial transit), inductances (tube inductance vs. static inductance) and impedances (stray resistance vs. electrical surge impedance). This leads to important information and insight into various aspects of the axial phase. In the present work we show that in a time-matched plasma focus shot we deduce the value of axial phase

current fraction  $f_c$  simply by measuring the calibrated voltage waveform and the uncalibrated current waveform. The scaling parameters  $\beta$  and  $\delta$  are fixed; and by form-fitting the measured current waveform to the normalised current waveform using the value of  $\alpha$  of the shot is determined uniquely; from which the peak current and the ratio of peak to average speed [the speed form factor (SFF)] are obtained. The average transit speed is measured by time-of-flight using the voltage upturn as indicator of end of axial phase. Then the SFF yields the peak speed. The measured voltage (back EMF), peak current and peak axial speed (all at the end of axial phase) allows the unambiguous measurement of  $f_c$ . The value of the mass swept-up fraction  $f_m$  is deduced from  $\alpha$  which is the ratio of the characteristic discharge and the characteristic transit times, both deduced during the non-dimensionalisation of the equations. Analysis of a time-matched shot in the INTI PF at 15 kV, 3 Torr D<sub>2</sub> gave  $f_c = 0.68$  and  $f_m = 0.05$ .

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## Introduction

Non-dimensionalising a system of equations is an important technique to obtain characteristic properties of the PF system described by the equations. The normalising or non-dimensionalisation also reduce the parameters of the system to a few scaling parameters [1]. A set of scaling parameters uniquely determines the behaviour of the system.

We apply this important technique to the axial phase of the Mather plasma focus which typically is controlled by

parameters: static inductance  $L_0$ , capacitance  $C_0$ , stray resistance  $r_0$ , anode and cathode radii  $a$  and  $b$  respectively (or ratio  $c = b/a$ ), anode length  $z_0$  and operational parameters including voltage  $V_0$  and gas pressure and gas type (or density  $\rho_0$ ) and also current drive fraction  $f_c$  and mass swept-up factor  $f_m$  [2–5].

In normalising the two coupled equations of motion and circuit for the plasma focus axial phase we reduce the parameters to just three scaling parameters  $\alpha$ ,  $\beta$ ,  $\delta$  which are the ratios of characteristic times (electrical discharge vs. axial transit), inductances (tube inductance vs. static inductance) and impedances (stray resistance vs. electrical surge impedance) [2, 3]. For each set of  $\alpha$ ,  $\beta$ ,  $\delta$  there is a unique set of current waveform and trajectory and speed profiles.

To demonstrate the use of this technique, we use the results to analyse an experiment in which we measure the calibrated voltage waveform and the uncalibrated current waveform. From these waveforms we use the characteristic behaviour of the normalised equations and deduce  $f_c$  and  $f_m$ .

The current drive fraction and mass swept-up fractions are crucial in modeling the plasma focus. There are two pairs: one for the axial phase and one for the radial phase. The present work is discussing the axial phase model parameters.

Experiments had been carried out from time to time by several groups of researchers to determine the fraction of the circuit current flowing into the focus tube and taking part in the driving action on the plasma. The value which is generally accepted is in the range of 0.6–0.7. A method used to measure the plasma current is by placing magnetic probes to detect the magnetic field as the plasma sheath sweeps over the probes [6, 7]. The probe perturbs the plasma changing the flow field and magnetic field patterns sometimes completely changing the flow characteristics especially in smaller plasma focus devices. This method has recently been complemented by using current fitting [4, 8]; to get the values of  $f_m$  and  $f_c$  by iterative adjustments to the values of  $f_m$  and  $f_c$  until the axial phase parts of the current traces fit. However the effects of the variations of  $f_m$  and  $f_c$  are interactive so there is always some uncertainty in this method of fitting [8–11]. A reference value of  $f_c = 0.7$  is generally assumed; allowing us to simplify the fitting procedure to a variation of  $f_m$  to produce the fit for the axial phase. Then typically only a minor tuning of the value of  $f_c$  (about the value of 0.7) is all that is necessary to produce a good unambiguous fit. So far all the fitting examples have confirmed that using  $f_c = 0.7$  as a starting point is consistent with the intensive and extensive study of experimental data for a large range of machines [12–17]. Nevertheless a method of independently finding  $f_c$  (without simultaneous adjustments to  $f_m$ ) is needed to verify the

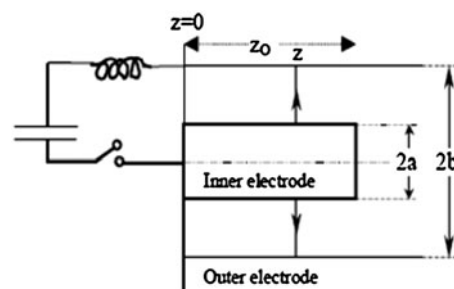


Fig. 1 Schematic of the axial phase of the Mather plasma focus

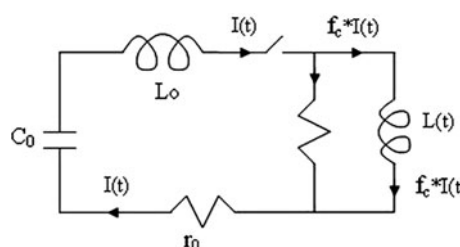


Fig. 2 Plasma focus circuit in the later part of the axial phase: the coaxial tube of the plasma focus up to the axially moving current sheet is treated as a time-dependent inductance  $L(t)$  or  $L_r$ , neglecting the plasma resistance  $r_p$

practice of assuming a reference value of  $f_c = 0.7$ . Through non-dimensionalising the two coupled equations describing the axial phase we deal essentially with one parameter  $\alpha$  which allows us to find  $f_c$  independently.

### Theory

#### Two Coupled Equations (Motion and Circuit) Describing the Axial Phase

We represent the axial phase of the plasma focus by the schematic shown in Fig. 1. A thin current sheet is driven by the  $J \times B$  force from position  $z = 0$  at  $t = 0$  to position  $z$  at time  $t$ . We use a modified snow-plow model in which the current sheet carrying a fraction  $f_c$  of circuit current  $I$  at position  $z$  has swept up a fraction  $f_m$  of the mass it has encountered.

Equating the rate of change of momentum at the current sheet to the driving force averaged across the coaxial channel we have the equation of motion as follows [2, 3]:

$$\rho_0 \pi (c^2 - 1) a^2 f_m \frac{d}{dt} \left( z \frac{dz}{dt} \right) = \frac{\mu f_c^2}{4\pi} (\ln c) I^2 \quad (1)$$

$$\therefore \frac{d^2 z}{dt^2} = \left[ \frac{f_c^2}{f_m} \frac{\mu (\ln c)}{4\pi^2 \rho (c^2 - 1)} \left( \frac{I^2}{a} \right)^2 - \left( \frac{dz}{dt} \right)^2 \right] / z$$

Plasma focus axial phase circuit is shown in Fig. 2. The fraction  $f_c$  of circuit current  $I$  flows in the current sheet with

the rest of the current rendered ineffective shown as flowing through an unlabelled leakage resistance. In the high Magnetic Reynold’s number regime [18, 19] where the electromagnetic drive predominates we may ignore the plasma resistance; and the circuit equation may be written as (with the inductance of the coaxial tube  $L_t = [\mu/(2\pi)] \ln(b/a)z$  where  $\mu$  is the permeability of the plasma):

$$\begin{aligned} \frac{d}{dt} [(L_o + Lf_c)I] + r_o I &= V_o - \int \frac{Idt}{C_o} \\ \frac{dI}{dt} &= \left[ V_o - \frac{Idt}{C_o} - r_o I - If_c \frac{\mu}{2\pi} (\ln c) \frac{dz}{dt} \right] / \left[ L_o + \frac{f_c \mu}{2\pi} (\ln c) z \right] \end{aligned} \tag{2}$$

Equations (1) and (2) are the coupled generating (or governing) equations of this model with interactive effects of the current I and the position z of the current sheet.

Non-dimensionalisation of the Two Coupled Equations

To characterise this set of coupled equations we normalise the equations to obtain the relevant scaling parameters. We replace variables t, z, I by non-dimensionalised quantities as follows:

$$\tau = t/t_0, \zeta = z/z_o, \iota = I/I_0$$

where the normalizing quantities  $t_0$ ,  $I_0$  and  $z_o$  are carefully chosen to be relevant, characteristic, convenient quantities, reflecting the physics of the problem i.e.,  $t_0 = \sqrt{L_o C_o}$ ,  $z_o =$  length of anode and  $I_0$  is  $V_o/Z_o$  where  $Z_o = \sqrt{L_o/C_o}$  is the surge impedance (noting that  $I_o$  is the peak current of the  $L_o$ - $C_o$  discharge circuit with capacitor  $C_o$  charged initially to  $V_o$  and with  $r_o = 0$ ).

After non-dimensionalising the Eqs. (1) and (2) may be written as follows:

$$\frac{d^2 \zeta}{d\tau^2} = \frac{[\alpha^2 \iota^2 - (\frac{d\iota}{d\tau})^2]}{\zeta} \tag{3}$$

and

$$\frac{d\iota}{d\tau} = \left( 1 - \iota d\tau - \beta \iota \frac{d\zeta}{d\tau} - \delta \iota \right) / (1 + \beta \zeta) \tag{4}$$

where  $\alpha = (t_0/t_a)$  with  $t_a$  identified as the characteristic axial transit time:

$$t_a = \left[ \frac{4\pi^2 (c^2 - 1)}{\mu \ln c} \right]^{1/2} \frac{\sqrt{f_m}}{f_c} \frac{z_o}{(I_o/a)/\sqrt{\rho_o}} \tag{5}$$

$$\beta = \frac{L_a}{L_o} \tag{6}$$

$$\delta = r_o/Z_o \tag{7}$$

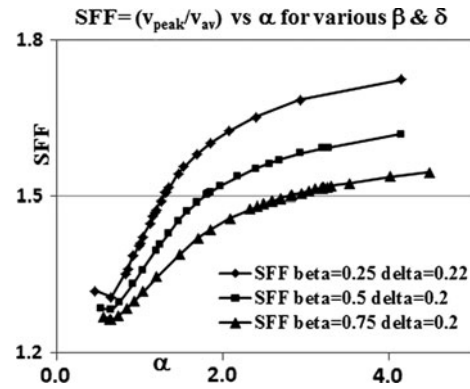


Fig. 3 SFF versus  $\alpha$  for various combinations of  $\beta$  (beta) and  $\delta$  (delta)

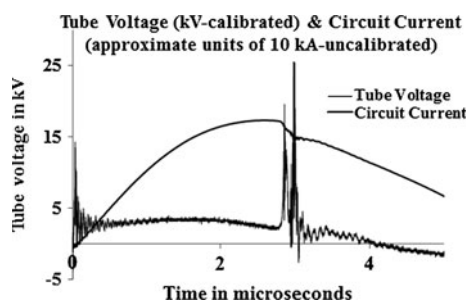
where  $L_a = \frac{f_c \mu}{2\pi} (\ln c) z_o$  is the inductance of the plasma focus tube when the current sheet has reached the end of the anode at  $z = z_o$ .

Some Characteristics of the Electro-dynamics: Uniqueness of Current Profile, Trajectory and Speed Profile of Current Sheet—The Speed Form Factor SFF

Equations (3) and (4) are the non-dimensionalised equivalent of governing Eqs. (1) and (2). Whereas Eqs. (1) and (2) have the parameters b, a,  $f_c$ ,  $f_m$ ,  $\rho_o$ ,  $V_o$ ,  $L_o$ ,  $C_o$  with no simple behavioural pattern discernable; in the normalised form Eqs. (3) and (4) can be seen to have behaviour dependant on three ratios of characteristic times ( $\alpha$ ), inductances ( $\beta$ ) and impedances ( $\delta$ ).

For each set of  $\alpha$ ,  $\beta$  and  $\delta$  the form of the equations tell us that the profile of current versus time is uniquely different from any other set; as are the profile of distance versus time and speed versus time. This leads to practical applications. As an example we look at the speed form factor (SFF), defined as the ratio of peak axial speed over the average axial speed. Figure 3 shows the SFF as a function of  $\alpha$  for three values of  $\beta$  at  $\delta = 0.2$ .

The value of  $\delta = 0.2$  is chosen ( $\delta = 0.22$  for the case of  $\beta = 0.25$  to suit the INTI PF device); as most modern capacitor banks have such a low inherent damping factor. The range of  $\beta = 0.25$ – $0.75$  covers the range of existing typical machines. The range of  $\alpha$  (0.4–4) covers operation of machines reaching the end of axial phase from very early in the discharge (way before the time of peak current) to very late in the discharge (way after time of peak current). Thus Fig. 3 is a valuable tool providing the SFF which allows the peak axial speed to be obtained just from the average transit speed measured by dividing  $z_o$  by the time of the axial phase taken from  $t = 0$  to time of start of the rollover of the current into the dip (or the start of the



**Fig. 4** Time-matched shot in INTI PF at 15 kV 3 Torr D<sub>2</sub>. The current trace is the smoother trace and is uncalibrated in current amplitude though we have given it an approximate scale of 10 kA per vertical unit

upturn into the voltage spike, see Fig. 4). Moreover for each set of  $\alpha$ ,  $\beta$  and  $\delta$  the peak current is also found in non-dimensionalised form which could be immediately converted into the real current by multiplying by  $I_0$ .

### An Application: Obtaining the Drive Current Fraction $f_c$ of the Axial Phase

We now use this information to obtain the value of  $f_c$  for the axial phase of a plasma focus. The value of  $f_c$  is an important parameter in the study of electromagnetic shock tubes and the plasma focus alike. Its importance for numerical models has already been discussed in an earlier paragraph as has the importance of verifying its value using a method independent of interaction with  $f_m$ .

In this present work we measure the tube calibrated voltage  $V$  (inductive back EMF) at the input flanges of the plasma focus together with the uncalibrated current waveform. We choose a time-matched shot in which the end of the axial phase occurs at the time of peak current. For the same shot the average speed of the axial current sheet is measured by time-of-flight and the peak (end axial) speed  $(dz/dt)_{peak}$  is obtained by multiplying by a speed form factor obtained from Fig. 3.  $I_{peak}$  is also found from the relevant non-dimensionalised current versus time curve. From these measured values of  $V$  (at time  $I_{peak}$ ),  $I_{peak}$  and  $(dz/dt)_{peak}$ , the value of  $f_c$  is determined. The analysis is as follows.

### Analysis of the Problem

The circuit of the plasma focus is represented in Fig. 2. The current  $I$  enters the plasma focus tube. A fraction  $f_c$  (to be determined) of this current creates a current sheet which, interacting with its own magnetic field is accelerated down the coaxial tube, represented in the figure by the inductance  $L(t)$ . A leakage current flows across the back wall region of

the tube and does not take part in the current sheet action. This is represented in the figure by an unlabelled resistance. As the shock wave system develops moving faster and faster, the plasma conductivity rises, the electrical resistance of the current sheet drops. We consider the later part of the axial phase when the axial current sheet has been accelerated to a high speed attaining a high magnetic Reynold's number at which the electromagnetic approximation  $r_p I_p(t) \ll d(L(t)I_p(t))/dt$  may be applied. We represent the plasma current as  $I_p = f_c I(t)$ .

The voltage probe has its high voltage point attached to the anode collector plate and its low voltage point connected to the cathode collector plate of the INTI PF; measuring the voltage  $V$  across  $L(t)$  also written as  $L_t$  we have:

$$V = d(L(t) I_p(t))/dt$$

$$V = d(L_t I_p)/dt = f_c d(L_t I)/dt = f_c I (dL_t/dt) + f_c L_t (dI/dt)$$

A plasma focus particularly in deuterium, is typically operated with the end of the axial phase occurring at or near peak current so that the radial phase produces the most energetic compression. We may call this mode of operation time-matched. Moreover the current peak is typically relatively gentle so that the condition  $dI/dt = 0$  occurs for such time-matched shots. As mentioned earlier we are choosing such a time-matched shot where :

$$V = f_c I (dL(t)/dt)$$

For the coaxial geometry of the INTI PF (see section below for INTI PF parameters):

$$L_t = [\mu/(2\pi)] \ln(b/a) z \text{ where } \mu = 4\pi 10^{-7}; b/a = 3.2/0.95$$

So that  $L(t) = 2.4 \times 10^{-7} z(t)$  and for the time-matched situation the current and the axial speed are respectively  $I_{peak}$  and  $(dz/dt)_{peak}$ ; hence:

$$V = f_c \times I_{peak} \times 2.4 \times 10^{-7} \times (dz/dt)_{peak} \tag{8}$$

### The Experiment

For this work we use the INTI PF which is one of the UNU ICTP PFF network machines [18–22]. The configuration parameters are as follows:

- Static inductance  $L_0 = 114$  nH,
- Capacitance  $C_0 = 30$   $\mu$ F,
- Circuit resistance  $r_0 = 13$  m $\Omega$ ,
- Cathode radius  $b = 3.2$  cm,
- Anode radius  $a = 0.95$  cm,
- Anode length  $z_0 = 16$  cm,
- Voltage  $V_0 = 15$  kV, Gas pressure = 3 Torr D<sub>2</sub>.

From Eq. (8) we see that to calculate  $f_c$  we need the voltage  $V$ , the value of  $I_{\text{peak}}$  and the value of  $(dz/dt)_{\text{peak}}$  at the time of peak current which corresponds to the end of the axial phase.

For voltage measurements we use a Tektronix high voltage probe P6015A with a rise time of 8 ns and an attenuation factor of 1,000 times. For current measurements we use a 7-turn Rogowski coil which picks up the  $dI/dt$  which we then integrate numerically to obtain the current trace [23]. We measure only the current waveform, noting that the calibration constant of a current coil is not very reliable so that we prefer in this experiment to obtain the  $I_{\text{peak}}$  from the appropriate computed  $i$  versus  $\tau$  curve chosen by the appropriate  $\alpha$ ,  $\beta$  and  $\delta$  combination of the shot. For the value of  $(dz/dt)_{\text{peak}}$  we first measure the average transit speed of the current sheet by dividing the anode length  $z_0$  by the transit time measured from the start of current to the start of the roll-over of the current into the current dip. Then we multiply this average speed by the SFF of the appropriate  $\alpha$ ,  $\beta$ ,  $\delta$  of the shot.

The scaling parameters  $\beta$  and  $d$  for the INTI PF are calculated from known parameters as  $\beta = L_a/L_0 = 0.25$  and  $\delta = r_0/(L_0/C_0)^{0.5} = 0.22$ .

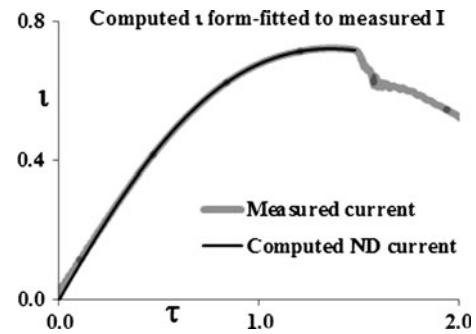
We cannot calculate the primary scaling parameter  $\alpha$  since the transit time contains the parameter  $f_m$  and  $f_c$  which we are trying to find [see Eqs.(4) and (5)]. However we have the measured time profile of the current and we may fit this measured time profile to the various non-dimensionalised current waveforms generated by varying  $\alpha$ ; with  $\beta$  and  $\delta$  fixed at 0.25 and 0.22 respectively. Only that current waveform with the correct  $\alpha$  will fit the measured current waveform.

Once we have determined the  $\alpha$  of our shot we have the appropriate SFF from Fig. 3 from which we deduce  $(dz/dt)_{\text{peak}}$  using our measured average speed. Also the fitted non-dimensionalised current waveform gives us the appropriate non-dimensionalised current peak from which  $I_{\text{peak}}$  may be obtained. We have the value of tube voltage  $V$  measured with the voltage probe.

Then using Eq. (8) we calculated  $f_c$ ; and then using Eq. (5) we deduce  $f_m$ .

### The Results

Figure 4 records the voltage waveform in kV of a time-matched shot in INTI PF at 15 kV 3 Torr  $D_2$ . In the same trace is recorded the numerically integrated current waveform from a 7-turn Rogowski-type coil operated as a  $dI/dt$  detector. We have found that this coil has sufficiently wide frequency response to faithfully record a plasma focus current waveform [23]. We have a calibration constant for this coil, but for this experiment, as already explained



**Fig. 5** Fitting the computed non-dimensionalised (ND) current  $i$  waveform to the measured current waveform  $I$  by varying  $\alpha$  until a good fit is obtained. The best fit (shown in this Figure) was obtained at  $\alpha = 1.53$  (with fixed  $\beta = 0.25$  and  $\delta = 0.22$ ). The scales are the computed values of  $i$  and  $\tau$

earlier we prefer to obtain the  $I_{\text{peak}}$  from the non-dimensionalised current trace of the appropriate  $\alpha$ ,  $\beta$ ,  $\delta$  combination.

Using the measured current waveform (appropriately scaled) as a reference waveform, a non-dimensionalised current waveform is generated with  $\beta = 0.25$ ,  $\delta = 0.22$  and  $\alpha$  varied from 0.5–2.0.

Each generated waveform is compared by overlaying to an appropriately time- and amplitude-scaled version of the measured current waveform. It was found quickly that for our shot the ranges from  $\alpha = 0.5$ –1.4 and from 1.7–2.0 shows no possibility of any fit.

The range of  $\alpha = 1.4$ –1.7 was investigated carefully and a good fit was found in the range of  $\alpha = 1.49$ –1.58. We decided on an a value of  $1.53 \pm 0.05$  giving (see Fig. 3) a value of SFF range of  $1.546 \pm 0.01$ .

The case for  $\alpha = 1.53$  is shown in Fig. 5. The small discrepancy near time  $\tau = 0$  is always observed in such fitting between measured and computed current traces, due to the imperfect switching and added resistance in the early stages of the discharge which is not modelled. The fit after  $\tau = 0.2$  and going on to the end of the axial phase is very good.

From this fit we obtain the peak value of  $i$  as 0.72 which on un-normalising to  $I_0 = 243$  kA gives a value of  $I_{\text{peak}} = 175$  kA.

From the start of the upturn of the voltage trace (Fig. 4) at time  $t = 2.65 \mu\text{s}$  (confirmed by the start of the roll-over of the current trace) we obtain the average transit speed as  $6.04 \text{ cm}/\mu\text{s}$  which after multiplying by  $\text{SFF} = 1.546$  gives  $(dz/dt)_{\text{peak}} = 9.3 \text{ cm}/\mu\text{s}$ .

From Fig. 4 we measure the voltage value corresponding to the time of  $(dz/dt)_{\text{peak}}$  as 2.65 kV.

Thus from Eq. (8) we obtain  $f_c = 2,650/(175 \times 10^3 \times 2.4 \times 10^{-7} \times 9.3 \times 10^4) = 0.68$ .

Then from Eq. (5) we use this value of  $f_c$  to compute  $f_m = 0.05$ .

## Conclusion

In this paper we obtained the two coupled equations of motion and circuit describing the trajectory and current profile of the radial phase of the Mather-type plasma focus. We normalised these equations and obtained two coupled equations the behaviour of which depends on three scaling parameters  $\alpha$ ,  $\beta$  and  $\delta$  which are ratios of electrical to transit times, inductances and impedances respectively. Thus the behaviour of the axial phase of any plasma focus may be characterised by these 3 parameters. A corollary is that the normalised current waveform, trajectory and speed profile are unique for each combination of  $\alpha$ ,  $\beta$ ,  $\delta$ . This could lead to important information and insight into various aspects of the axial phase of the plasma focus. As an example we show that in a time-matched plasma focus shot we are able to deduce the value of axial phase current fraction  $f_c$  simply by measuring the calibrated voltage waveform and the uncalibrated current waveform. The scaling parameters  $\beta$  and  $\delta$  are fixed for any plasma focus and by form-fitting the measured current waveform to the normalised current waveform, the value of  $\alpha$  of the shot is determined uniquely; from which the peak current and SFF of that shot are obtained. The average axial transit speed of the shot is measured by time-of-flight using the current rollover or voltage upturn as indicators of the end of the axial phase. Then the SFF yields the peak speed. The measured voltage, the peak current and the peak axial speed (all at the corresponding time of the peak current, peak axial speed and arrival of current sheet at the end of the anode) allows the measurement of  $f_c$ . Once  $f_c$  is measured the value of  $f_m$  is then deduced from the value of  $\alpha$  being the ratio of the characteristic discharge time and the characteristic transit, both of which expressions are deduced during the normalising of the equations. Analysis of a time-matched shot in the INTI PF at 15 kV, 3 Torr D<sub>2</sub> gave a result of  $f_c = 0.68$  and  $f_m = 0.05$ . This determination of these model parameters, especially  $f_c$ , using a method which unambiguously fixes the value of  $f_c$  is of crucial importance: generally for the fitting of current curves in modelling and particularly for the recent development of novel plasma focus circuits and ideas such as current-steps [24, 25] radiative cooling and collapse [26] and obtaining reference numbers and scaling trends for fast ion beams and fast plasma streams [27, 28].

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