# APPLICATION OF FUZZY LOGIC TO DIAGNOSTIC TESTING IN AN E-LEARNING ENVIRONMENT

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## ABSTRACT

In an e-Learning environment, before a student moves on to the next module a diagnostic test is given from which a grade is provided. However, it is far more useful to the student if he is given some indication as to his actual capability in relation to the instructor's expectations. A letter grade or a numerical score (except at the extreme ends of the range) while desirable, does not provide sufficient insight into how the student fairs. What is needed is feedback to the student that helps him in his continual improvement in the subject matter. It is the myriad of variables such as, the student's speed in working out the problems, guessing, and the fraction of correctly answered problems. A careful consideration of these new variables gives this investigation a fresh outlook to the problem of diagnosis that has not been previously addressed. This article presents an application of Fuzzy Logic to diagnose a student's mastery of a narrow sliver of subject matter that he has studied. Recommendations are then given as to how he should proceed with his studies. The power of this approach lies in the ability of the computer administrating the diagnostic test to track a student's problem-solving speed for each and every problem that is attempted. Interestingly it is also possible to determine if the student made guesses instead of actually working out the correct answers to the problems. For diagnostic purposes, a correct answer from exercising a guess should not count towards the number of correct answers but as an unanswered question. Such a capability is not available for in-class tests because the actual answering speed for each question is not tracked, nor is it possible to determine if a correctly answered problem comes from guessing at the answer. This article describes the workings of a Fuzzy Logic diagnostic program and how it addresses all these interesting variables and incorporates them into a useful tool for assisting e-learning students in determining their mastery of the subject matter.

## KEYWORDS

Fuzzy logic, e-Learning, Diagnostic testing

## INTRODUCTION

In the past decade there has been a rapid increase in the use of information and communication technologies for instruction at many educational institutions. Recent years have also seen an exponential rise in the educational cost structure of many such institutions. It is generally perceived that such costs can be contained with the application of the aforementioned technologies to teaching and learning. In actual fact, application of these technologies has resulted in more than just increases in efficiencies (and therefore containment in costs). There have been improvements in quality, teaching and learning effectiveness, knowledge assessments, as well as in the breadth and speed of educational dissemination.

One of the areas of interest to the current investigators is in the area of diagnostic testing. In an introductory quantitative course such as Statistics, Calculus, Linear Algebra, or even Differential Equations, the size of such classes are generally large. Much time and energy have to be expended on the part of the instructor to generate and give in-class assessments, as well as valuable class time used up in determining student mastery of the subject matter. In an e-learning environment, however, tests can be given very soon after students have been presented the material to ensure that they have acquired the mechanics of solving a particular type of problem just introduced. The purpose of the diagnostic test, therefore, is not to assign grades to students, but to provide them with quick and timely feedback on their level of understanding and their ability to solve related problems. Clearly, such tests delivered on-line and graded automatically, can be repeatedly given to students at their convenience. They do not have to wait until a given date and time to take the test in a particular setting as in-class students do, nor are grades or points given that stays on record. The existence of extensive repositories of problems facilitate the process of problem generating each time the testing procedure is invoked so that a student can be tested again almost right away if he had not done well in prior tests on the same material. This approach works well so long as no long-winded derivations are required as part of the test. Writing out equations in a virtual environment is still a challenge. (Caprotti, 2007, Maplesoft). At this point, it may still be more convenient and efficient for e-learning students in quantitative courses to work out the problems on paper and find from a set of choices, the correct answer (Nascy, 2004).

## FUZZY LOGIC

Lofti Zadeh, the founder of fuzzy logic, contends that a computer cannot solve problems as well as humans unless it is able to duplicate the imprecision in the thinking characteristics of a human being. Very often, we rely on fuzzy expressions such as "often," "very good," or "tall" while a computer current is limited to true-or-false, everything-or-nothing, which are crisp modes of logic. Interestingly, this idea of fuzziness has actually taken root. Over the past four decades, fuzzy logic has actually blossomed in the quantitative fields of engineering (Ross, 2010) as well as in business and finance (Von Altrock, 1997), fields that one would think need precision. So would it be when one thinks of assessments. The idea of pairing fuzziness with testing, as the investigators are trying to present in this article will, prima facie, also appear to be ridiculous.

There is nothing fuzzy about fuzzy logic. It is actually a quantitative science, and is fuzzy in name only. What fuzzy logic is able to handle is the imprecision that we humans use on a daily basis and yet, not only are we able to function with such imprecision, we thrive on it. Take for as example of a student being given a grade of 75%. To an elementary school student in arithmetic that would have been a failure in the test, but to a postgraduate student in Advanced Dynamics, that might have been an excellent grade! So, a crisp numerical number does not necessarily mean the same thing to everyone, nor would it even mean the same thing for different postgraduate courses given by different instructors. In a similar way, fuzziness would also be imprecise, just as how one would consider another as "rich," vary widely among different peoples in different parts of the world.

The primary objective of this article is not fuzzy logic, but is in the use of fuzzy logic for diagnostic assessment. For a more in-depth understanding of fuzzy logic, the reader is encouraged to look into introductory texts such as (Chen, 2001). This article will refrain from getting engrossed in the mechanics of mathematical manipulations in fuzzy logic, but instead will concentrate on using the Fuzzy Logic Toolbox, a set of routines (Mathworks, 2010), written in Matlab. This toolbox enables the user to go through the process of defining membership functions for the various crisp inputs (a process called fuzzification), the rule-base for processing the resulting fuzzy inputs, and the defuzzification of the outputs. The basic idea is this: inputs to the fuzzy logic program are processed though a sequence of rules that govern the interactions between the inputs. The output gives the results that one is seeking. In subsequent sections, we will examine how diagnostic testing can be handled with fuzzy logic.

## DIAGNOSTIC ASSESSMENT

While testing is not necessarily the best way (Toby, 2004) to determine knowledge acquisition, it is the most widely used and probably the most efficient approach. In a virtual environment, it is the primary approach to determine student mastery of the subject matter (Barbosa, 2005; Booth, 2003; Encheva, 2005; Herskowitz, 2004), though there has been some variations to the approach (Meijer, 2002). Much work has also been published on implementations of online programs for testing (Booth, 2003; Jantschi, 2004). However, all these articles have concentrated on testing for grade allocations rather than on diagnostic testing. While both these types of tests hold many characteristics in common, there is a slight difference in the objective of these tests. One common characteristic is the need to determine the level of mastery of subject matter. In the case of assessment with grade allocation, points or a letter grade is given. This is not so with diagnostic assessment - the objective is to provide valuable and timely feedback to the student in an e-learning environment on the level of achievement he has attained before going on to the next topic or module. No letter grade is given. Instead, advice is provided to the student as to his area of weakness, the need to review or to work out more problems of a particular type, or even to revisit the entire module all over again if the student did very poorly in relation to the *expectations* of the instructor (as opposed to one that is in relation to the level of accomplishment of the rest of the class when a letter grade is given). In the case of a student being diagnosed as weak, he will have to take the advice given by the diagnostic test, and retake a different version of the diagnostic test again until he has been given a clean bill of passage to continue on to the next level. That is the crux of the difference between these two types of assessments.

One advantage of an online test that is not readily present in an in-class test is the availability of an overseer - the computer which is capable of tracking the time taken by the student in solving each problem. In-class tests only set an upper limit in the time available to the student to answer all of a given test, or section of a test, but it is not possible to ascertain how much time the student takes to answer any given question. From that time measure for each question, much information can be deduced as to how well the student is able to answer the given question. Of course, the level of difficulty of each question also affects the time taken to answer that particular question. This can be taken into account as well.

Note that as we use the qualifiers such as "difficulty" or "answering speed", the reader should keep in mind that such quantities are actually imprecise. How would one assign a value of "difficult" in a range of $1 \rightarrow 10$ (10 being most difficult)? Or for that matter, "quite difficult"? Or "fast" and "quite fast"? Any numerical assignment is a value judgment that is different for different instructors who assigns such values. The ease by which fuzzy logic is able to handle such imprecision permits one to work through the value judgments and fuzzy inference rules that produces at the output, advice that serve as valuable and timely feedback to the student.

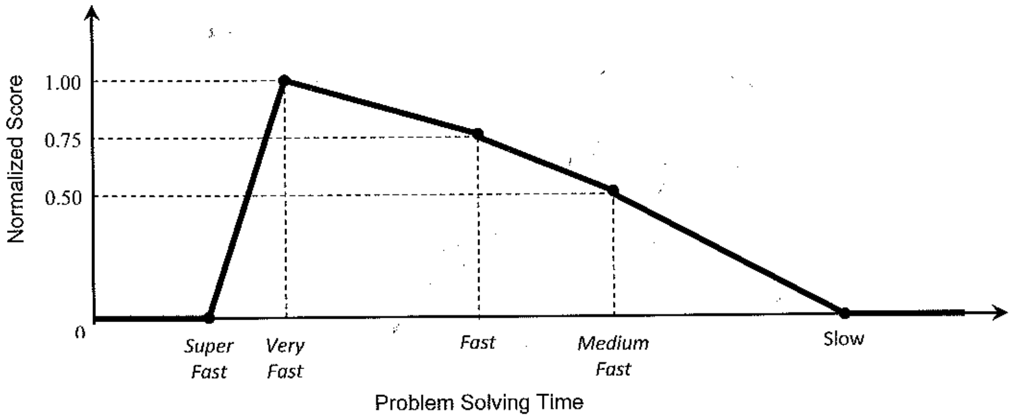


**Figure 1. Problem solving time score normalization for each problem.**

One way to handle the variation in difficulty of a question is to imbed this variable into the problem-solving time. This is the time taken to solve each problem, and since the test is taken on-line, it is possible to gather the data pertaining to the time taken for each problem that the student has attempted. Scores can then be assigned: "Very Fast," "Fast," "Medium Fast," and "Slow" such as that shown in Figure 1. Note that any question no matter how difficult or easy can be cast into a time axis for problem-solving time. It will be different for each question and it already accounts for the difficulty aspect of the question.

There is another aspect of this problem-solving time that is rather unique in its application. In the case of a student making a guess at the answer (since all problems are multiple-choice) without having spent the required minimum time to solve it, the problem-solving time will therefore be small. This means, for the purposes of diagnostic testing, even if a question is guessed correctly, the problem should not be counted as correct. There is no ethical issue with this action since no grade or points will be assigned. Note that if a problem is solved beyond the "Very Fast" level to a "Super Fast" level, it is presumed that there is a greater likelihood that the student is guessing the answer. For that, a lower score is given beyond the "Super Fast" level, it is presumed that the only way anyone can answer the question in such a short time, is by guessing. A score of zero is assigned denoting that the student does not know how to solve the problem, even if he had gotten it correct. In other words, if too little or too much time is used in working out a problem, it can be presumed that there is a lack of knowledge for that problem on the part of the student.

This approach is radically different from the commonly used method of assigning a penalty score to each problem that is incorrect, and the penalty score is then subtracted from the total score for all problems correctly answered. We have decided against using this penalty approach because what the diagnostic test is meant to do is to find out how much the student has actually mastered in his learning process. It should not be a matter of how well the student plays dice in eliminating some of the obviously incorrect choices for a given problem before randomly choosing whatever is left, thereby increasing his odds of getting more points.

Instead, we have arrived at the conclusion that there are three input variables for determining the student's level of subject mastery: the fraction of test questions the student have attempted correctly (but not by guessing), the variance and the mean of the problem-solving time. For each question that is correctly answered (including those that are presumably correct though guessing), the on-line program use the map shown in Figure 1 above to determine the score for that question. This is carried out for all the questions. From all the questions correctly answered, the Fraction Correct (ratio of the number of correctly answered questions to the total number of questions) is calculated. The standard deviation and mean of the problem-solving times for all correctly answered questions are also computed.

### *Fuzzy Inference System*

The Fraction Correct may be fuzzified into three groups: "High," "Medium" and "Low." For each of these groups, rules are constructed in conjunction with the other two variables "mean" and "standard deviation" of the problem-solving time. Again, the variables for "mean" and "standard deviation" for problem-solving time are also fuzzified into "High," "Medium" and "Low." These rules are shown in Figure 2. The fuzzified membership functions of "High," "Medium" and "Low" for the standard deviation for problem-solving time is illustrated in Figure 3.

The fuzzy rules listed in Figure 2 are then transcribed and entered into the Fuzzy Rule Base in the Fuzzy Logic Toolbox and a window of that is shown in Figure 4. An example is shown in which the variable: standard deviation is "low", the mean is "high" and the ratio (fraction correct) is "high" with the resulting performance of "Excellent." If the "mean" (of the normalized score) is "high", it means that the overall normalized score for all the correctly answered problems were completed from "fast" to "very fast." This implies that the student has a good mastery of the subject. The next variable is "ratio" and that is also "high" implying that the student has gotten a big proportion of the questions correct. Finally, the variable "standard deviation" is "low" implying that the student has been consistent in answering all the questions at a "fast" to "very fast" pace, with no slow performance nor was he guessing.

The result is "Excellent" which is what we specified as Rule #7 in the Rule Editor. Since there are three membership functions for each of the three variables, there will be a total of $3 \times 3 \times 3 = 27$ rules, although only 24 can be seen in the figure.

Figure 5 shows the window for the Rule Viewer. What this window shows are the membership functions that are fired (affected) by the values (0.2, 0.5, 0.5) for the three input variables and how the performance of 5.9 is calculated. Figure 6 illustrates pictorially that the three inputs "standard deviation", "mean" and "ratio" are fed into the Fuzzy Inference System (fis). The rule based is inferred by utilizing the Mamdani inference, resulting in the output called "Performance".

*1. Fraction Correct is High*

| | Mean | | |
|---|---|---|---|
| Std Deviation | High | Medium | Low |
| High | Good. Go to next module | More Practice Needed | More Practice Needed |
| Medium | Good. Go to next module | Good. Go to next module | More Practice Needed |
| Low | Excellent. Go to next module | Excellent. Go to next module | More Practice Needed |

*2. Fraction Correct is Medium*

| | Mean | | |
|---|---|---|---|
| Std Deviation | High | Medium | Low |
| High | Good. Go to next module | Practice More | Review & Practice More |
| Medium | Good. Go to next module | Practice More | Review & Practice More |
| Low | Good. Go to next module | Good. Go to next module | Review & Practice More |

*3. Fraction Correct is Low*

| | Mean | | |
|---|---|---|---|
| Std Deviation | High | Medium | Low |
| High | Review & Practice More | Redo Module in its entirety | Redo Module in its entirety |
| Medium | Review & Practice More | Review & Practice More | Review & Practice More |
| Low | Review & Practice More | Review & Practice More | Review & Practice More |

**Figure 2. Fuzzy rules relating the three Inputs in the fuzzy inference system.**

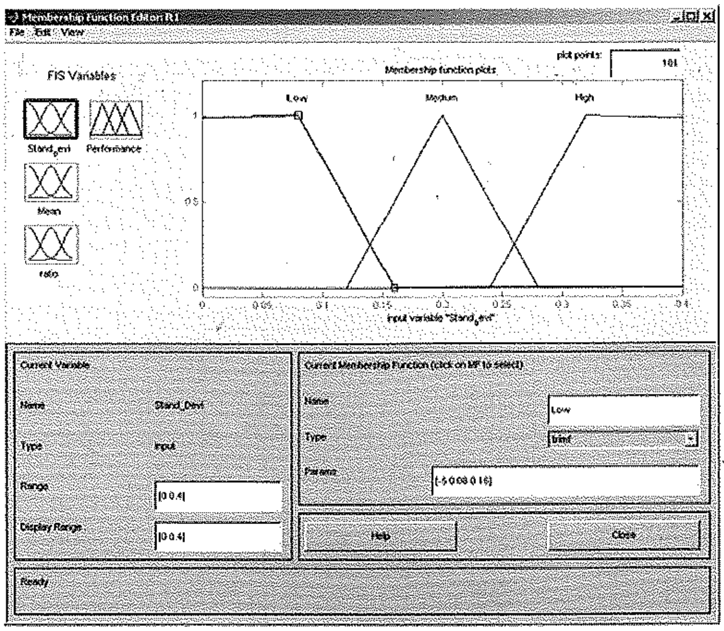


Figure 3. Membership function for standard deviation of problem-solving times (in Fuzzy Logic Toolbox).

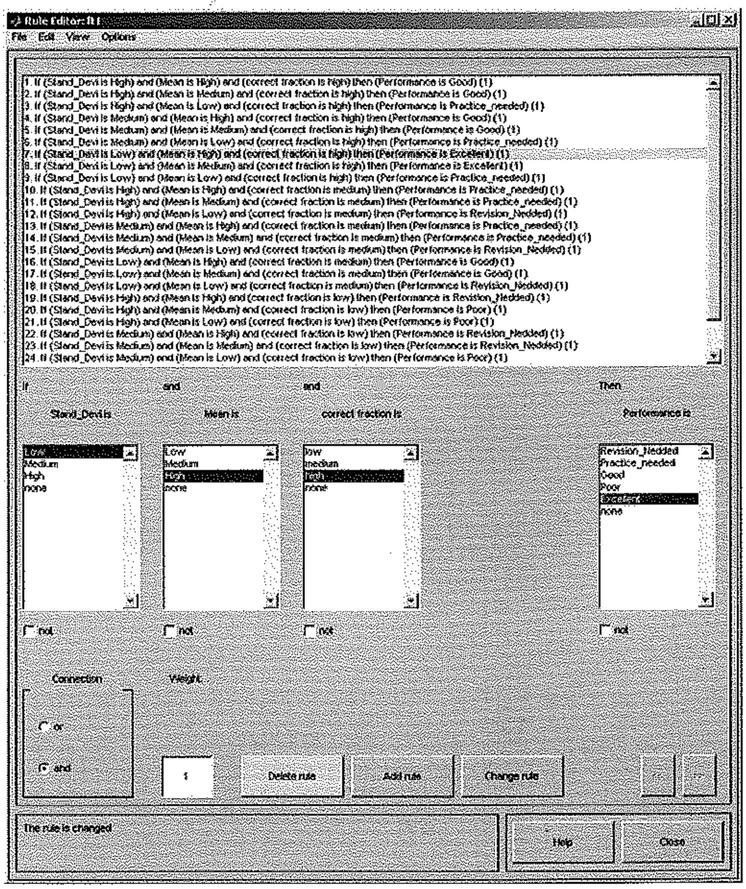


Figure 4. Rule Editor in Fuzzy Logic Toolbox.

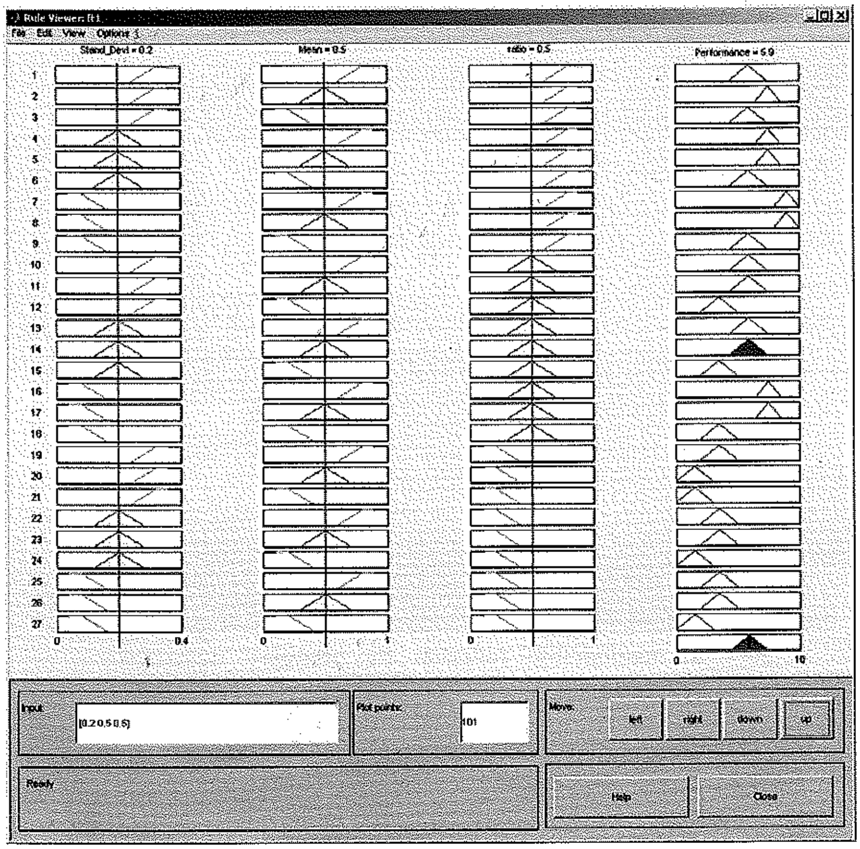


Figure 5. Rule Viewer in Fuzzy Logic Toolbox.

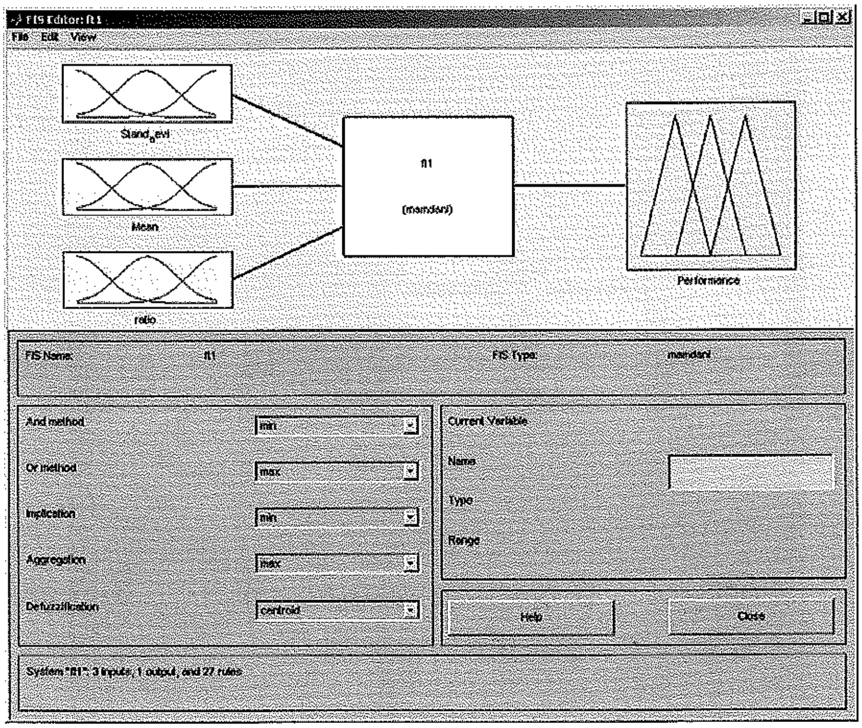


Figure 6. Fuzzy Inference System in Fuzzy Logic Toolbox.

## CONCLUSION

This article presents a novel method, one of applying fuzzy logic to diagnostic testing in an e-learning environment. The authors have also shown how imprecision actually works well in providing guidance to students taking diagnostic testing by leveraging the availability of time-tracking for each and every problem attempted by the student. This problem-solving time is than calibrated based on the difficulty of the problem and is normalized from 0 to 1 for every problem in the test. A side benefit of this problem-solving time capture is that it is now possible to identify to some degree of confidence that the student is guessing at the answers, instead of working them out. In the context of diagnostics, a guess can be considered equivalent to not knowing to the fullest extent how to answer that particular question.

By the application of rules relating to three input variables based on how the student answers the diagnostic test, it is then possible to provide a student as useful and timely feedback on their mastery of the subject material being tested. In the event that the student is found weak, he will be asked to review or even restudy the entire module, work out more problems before being allowed to take the diagnostic test again. When it is considered that the student has achieved sufficient mastery, he is then allowed to move on to the next module. No grade has been given since it is a diagnostic assessment.

In this investigation, the use of Mathwork's Fuzzy Logic Toolbox permits the authors to construct very quickly a framework using fuzzy logic to test out this concept for diagnostic test. There is then no need to develop the fuzzy inference engine and all associated programming that comes with implementing fuzzy logic into any given application. Further investigations will continue in terms of actually creating a diagnostic system that interfaces with current e-learning environment so that this concept can be incorporated seamlessly into that environment.

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