An Equivalent FIR Filter for an IIR Filter with Double Frequency Initialisation

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Indexing terms: *IIR, FIR, Transient Response, Initialization*

Abstract

IIR filters designed for steady state operation suffer from the transient effect caused by processing a finite number of samples. Their performance can be improved by using initialization techniques. This letter presents an equivalent FIR filter for an IIR filter operating in the transient and using double frequency initialization. This equivalent filter simplifies the transient analysis of IIR filters and provides a direct formula for calculating the transient frequency response and usable bandwidth.

Introduction

Digital infinite impulse response (IIR) filters are usually designed for steady state operation (i.e. processing a large number of input samples). If the number of processed samples is limited, such as in radar and sonar applications, then these filters will suffer from the transient effect. In this case, their frequency responses will be a function of the number of processed samples. These responses are called the transient frequency responses and they are different from the steady state responses.

One way of improving the transient frequency responses of IIR filters is by initializing their internal memories with values other than zeros. This requires an initialization processor to calculate the initial values from the first input sample in real time. These values will be the steady state values of the filter for a given input signal. The input signal may be approximated by a step function (step initialization) [1], or by a single tone sine wave (single frequency initialization) [2]. An initialization processor which can work at two frequencies (double frequency initialization) has also been introduced [3]. Other types of initialization for improving the transient performance of IIR filters exist [4, 5], however they are suitable for batch processing.

This letter derives an equivalent finite impulse response (FIR) filter for an IIR filter operating in the transient with double frequency initialization. The importance of this equivalent filter is that it simplifies the transient analysis of IIR filters with initialization by providing a direct formula for calculating the transient frequency response and usable bandwidth.

Double *frequency initialisation*

The transfer function of a real second order IIR notch filter is given by:

$$H(z) = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 - b_1 z^{-1} - b_2 z^{-2}}, \qquad (1)$$

where $\{a_1, a_2\}$ are the feed forward and $\{b_1, b_2\}$ are the feedback real coefficients of the filter. This filter has two complex conjugate poles p_1 and $p_2 = p_1^*$, and two complex conjugate zeros on the unit circle $z_1 = \exp(j2\pi f_1 T_s)$ and $z_2 = \exp(j2\pi f_2 T_s) = z_1^*$, where f_1 is the notch frequency, $T_s = 1/f_s$ is the sampling period, f_s is the sampling frequency, and $f_2 = f_s - f_1$.

The z-transfer function of this filter can be rewritten as a product of two first order complex sections:

$$H(z) = \frac{1 - z_1 z^{-1}}{1 - p_1 z^{-1}} \times \frac{1 - z_2 z^{-1}}{1 - p_2 z^{-1}}. \qquad (2)$$

The initialization processor, shown in Figure 1, calculates the initial values of the internal memories for each section from the first received input sample $x(0)$ and feeds them to the associated section. The output of each section is fed to a multiplier so that the filter will have two zeros, at the initialized frequencies, in the transient frequency response irrespective of the number of processed samples. The initial values of the internal memories of the complex sections are calculated as follows [3]:

$$m_1 = \frac{x(0)\, z_1^{-1}}{1 - p_1 z_1^{-1}}, \quad m_2 = \frac{x(0)\, z_2^{-1}}{1 - p_2 z_2^{-1}}. \qquad (3)$$

In order to obtain the transient response of the IIR filter with initialization, the in-phase and quadrature channels of the circuit in Figure 1 need to be simulated. In this letter, we derive an equivalent FIR filter that provides a simpler tool for analysing the transient response of IIR filters with initialization.

Equivalent filter

When processing N samples, the complex IIR section k ($k = 1$ or 2) in Figure 1 is equivalent to an FIR filter whose impulse response $\tilde{h}_k(n)$ is the same as the first N samples of the complex impulse response $h_k(n)$ of the IIR section. Thus:

$$\tilde{h}_k(n) = \{h_k(0), \cdots, h_k(N-1)\}, \tag{4}$$

When initialization is introduced, the impulse response of the equivalent FIR filter for the complex IIR section k processing N samples becomes:

$$\hat{h}_k(n) = \left\{ h_k(0), \cdots, h_k(N-2), \sum_{i=0}^{N-2} - h_k(i) e^{-j2\pi f_k T_s (i-N+1)} \right\}. \tag{5}$$

As can be seen from equation (5), the first $N-1$ samples will be the same as the samples of the original impulse response but the last sample will be different. This sample is responsible for forcing the frequency response of the equivalent filter to be zero at the initialized frequency. From Figure 1, the outputs of the two initialized complex sections are multiplied together to force the total frequency response to be zero at both frequencies (f_1, f_2). Therefore, the impulse response of the equivalent FIR filter will be the convolution of the two impulse responses:

$$h_{eq}(n) = \hat{h}_1(n) * \hat{h}_2(n) = \begin{cases} h(n) & , 0 \le n < N-1 \\ \sum_{i=0}^{N-1} \hat{h}_1(i) \hat{h}_2(n-i), & N-1 \le n \le 2N-2, \end{cases} \tag{6}$$

where $h(n)$ is the impulse response of the original second order filter in equation (1). The equivalent filter will allow the calculation of the transient response of the second order filter with initialization. The squared magnitude of the frequency response of the equivalent filter can be written as:

$$\left|H_{eq}(f)\right|^2 = d_0 + 2\sum_{i=1}^{2N-2} d_i \cos(2\pi f T_s i)\,, \qquad (7)$$

where $d_0 = \sum_{j=0}^{2N-2} h_{eq}^2(j)$ and $d_i = \sum_{j=0}^{2N-2-i} h_{eq}(j)\, h_{eq}(i+j)$.

Using equation (7) is simpler than simulating the in-phase and quadrature channels of the circuit in Figure 1 to calculate the transient frequency response of the filter. In addition, the useable bandwidth can easily be calculated from equation (7).

Example: Consider a second order Chebychev filter with the following z-transfer function:

$$H(z) = \frac{(1-(0.996-j0.0888)z^{-1})\,(1-(0.996+j0.0888)z^{-1})}{(1-(0.8111-j0.1727)z^{-1})\,(1-(0.8111+j0.1727)z^{-1})}. \qquad (8)$$

Table 1 shows the impulse responses of this filter when processing $N = 7$ samples only. It is clear from this table that the first 6 samples of the equivalent impulse responses of the two complex sections without $(\tilde{h}_1(n),\ \tilde{h}_2(n))$ and with $(\hat{h}_1(n),\ \hat{h}_2(n))$ initialization are the same. The only difference between the cases without and with initialisation is the 7th sample.

Similarly, the impulse response of the equivalent filter $h_{eq}(n)$ will have the same impulse response as the original real filter $h(n)$ in the first 6 samples. The samples between the 7th and the 13th will be different.

In order to demonstrate the usefulness of the derived equivalent FIR filter, equation (7) was used to analyze the transient response of the IIR when initialization is introduced. Figure 2 shows the squared magnitude frequency responses of the Chebychev filter without and with initialisation. As expected, with initialization the rejection characteristics of the filter are improved in comparison with the transient frequency response without initialization.

Conclusions

An equivalent FIR filter for a second order IIR filter operating in the transient with double frequency initialisation was derived. The equivalent FIR filter is applicable to both real and complex IIR filters. This equivalent filter simplified the transient analysis of IIR filters with initialization by providing a direct formula for the transient frequency responses and usable bandwidth. The equivalent filter will pave the way for optimising the transient characteristics of IIR filters.

References

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Figure and Table Captions

Figure 1. Two complex first order filters with their initialization processor.

Table 1. The impulse response of the Chebychev real filter and its equivalent filter when processing N=7 samples.

Figure 2. The squared magnitude frequency responses of the real second order Chebychev filter and its equivalent filter.

Figures and Tables

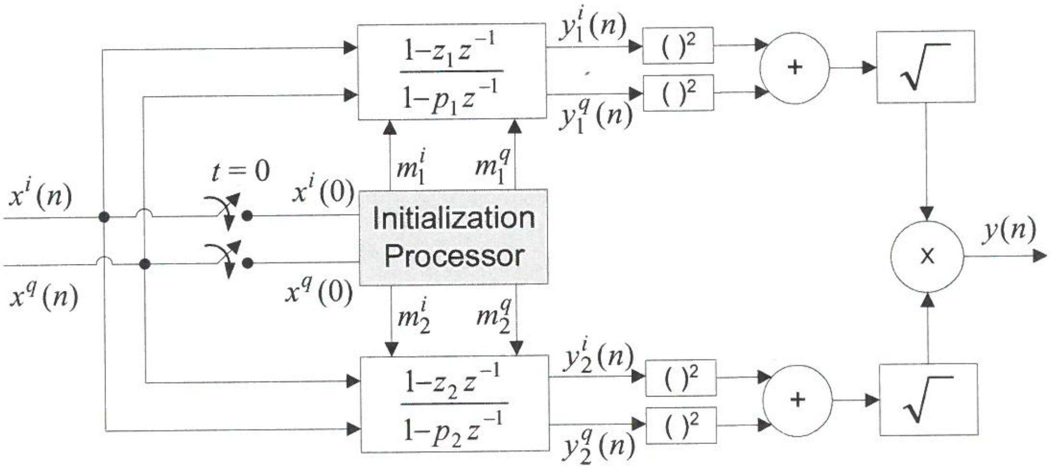
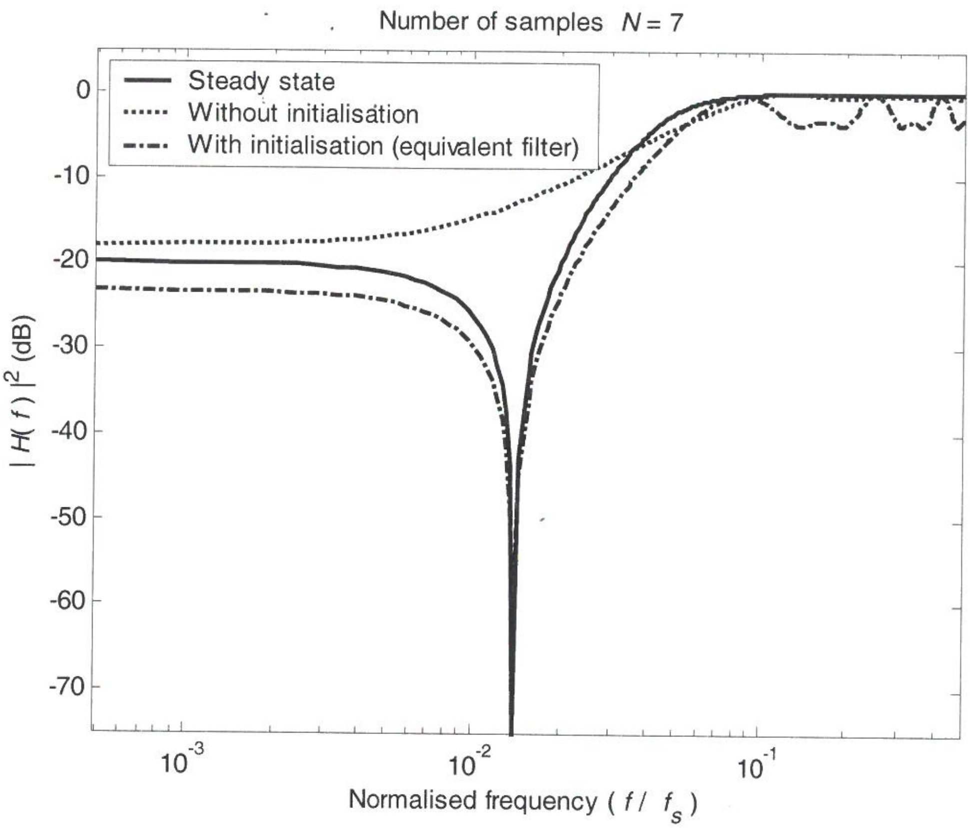


Figure 1. Two complex first order filters with their initialization processor.

Table 1. The impulse response of the Chebychev real filter and its equivalent filter when processing N=7 samples.

n	$h(n)$	$\widetilde{h}_1(n) = \widetilde{h}_2^*(n)$	$\hat{h}_1(n) = \hat{h}_2^*(n)$	$h_{eq}(n)$
0	+1.000	+1.000	+1.000	+1.000
1	−0.370	−0.185 + 0.084j	−0.185 + 0.084j	−0.370
2	−0.288	−0.165 + 0.036j	−0.165 + 0.036j	−0.288
3	−0.212	−0.140 + 0.001j	−0.140 + 0.001j	−0.212
4	−0.147	−0.113 − 0.023j	−0.113 − 0.023j	−0.147
5	−0.092	−0.088 − 0.039j	−0.088 − 0.039j	−0.092
6	−0.049	−0.065 − 0.047j	−0.165 − 0.356j	−0.248
7	+0.015			+0.059
8	+0.009			+0.066
9	+0.025			+0.067
10	+0.034			+0.063
11	+0.038			+0.056
12	+0.038			+0.154



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