

PLANAR B-SPLINE CURVE INTERPOLATION WITH VARIOUS PARAMETERISATIONS

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DECLARATION

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ABSTRACT

In previous days of shape modelling, the shape of engineering product are represented by points. It is will make life easier if could provide mathematical equations that able define the shape of an engineering product. The aim of the project is to interpolate the set of data points with B-Spline polynomial with different parameterization method such as chord length, centripetal and equal spaced parameterization. It is found that same set of data points, it may not be able to generate same shape of curve. The objective of this project investigate how the parameterization affect the shape of curve. In this paper, mathematical formula of different parameterization will be presented. The mathematical formula of different parameterization are then be convert into Excel Macro VBA to help generate results in a more efficiency way. Several set of data points which representing different characteristic of curve such as convex shape, inflected shape and Z shape are created to test how different parameterization on curve behave on it and the characteristic and properties of different parameterization are recorded. It was found that chord length parameterization curve are normally having bulges on longer chords, Centripetal parameterization curve following longer chords closer while for equal space easily to produce sharp points or loops with higher degree of curve. Furthermore, the impact of degree on different parameterization will also be discussed in this paper.

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DEDICATION

Dedicated to my beloved father, mother and brothers

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LIST OF ABBREVIATIONS

VBA	Visual Basic for Applications
CNC	Computer numerical control
CAGD	Computer Aided Geometric Design
2D	2 Dimension
3D	3 Dimension

NOMENCLATURE

<i>Symbol</i>	<i>Definition</i>
n	Degree of curve
k	order
C	Curve
$N_{i,p}$	Basis function
P	Control Point
u	Knot vector
p	Degree
Q_k	Points
u_k	Parameter value
U	Knot vector

CHAPTER 1

INTRODUCTION

1.1. Background

1.1.1. History

The numerical geometry was origin during the Second World War. A new design method are developed due to production pressure. The new design process are mainly graphical based on analytical curves and able to avoid tons of manual drafting work and required less time than formerly. After that, with grow of electronic computer, more technic are developed base on the traditional graphical methods. Previously, a process called lofting are used to define the cross-sections of a longitudinal curves. Lot of the new method dividing the surface and assembly to form a patches and those patches may be represented by mathematical formula (ID Faux, MJ Pratt,1979).

In 1963, Furguson develop 1 of the earliest patch system which using parametric rather than coordinates to define curve and surface. This patch system which able to twisted in three dimension has become the standard of usage then. Then the system were being invented to graphical display units, CNC machine tool and automatic drafting machines. When surface defining system became operation, number of important mathematical d developments occurred. First more properties of spline have been explored then a very basic theories of surface patches was evolved by Coons in year 1964. By now the system method is far removed from the origins and raised problems of how to decide freedom offered by modern methods of path. Then a new system call surface fitting which only required set of points of surface to define the curve are developed (ID Faux, MJ Pratt,1979).

1.1.2. Parameterization Methods

The parameterization of 2D or 3D data points is one of the fundamental steps in Computer Aided Geometric Design (CAGD) applications example: car design, aircraft design, texture mapping, remeshing, typography, and curve and surface fitting

The parameter value reflects the distribution of the given data points. A lot of research works have been done on parameterization method starting from the easiest method such as uniform, intermediate method, for instance, exponential, universal, and hybrid, until complex calculation such as rational chord length parameterization. The most common method used in solving the parameterization problems was centripetal method (H.Haron,2012). The most well-known parameterization methods nowadays are chord length, centripetal and equal spaced parameterization. These parameterizations is having its own advantage and disadvantage due to their properties.